Analysis of a Rounding Error

A rounding error can result when rounded (instead of pre-rounded) values are used for ensuing calculations. A rounding error exists when use of the rounded values gives an answer that is different from use of the pre-rounded values at a place in the answer that is a significant place. The George Washington Cause of Death homework assignment problems provide an instructive example. The rounding error could occur in question #3, but we need to look at questions # 1 & 2 first to get a thorough understanding.

*From question #1, re Vadakan’s estimate:*

\[
7.93 \text{ pt} \times \frac{1 \text{ qt}}{2 \text{ pt}} \times \frac{0.9464 \text{ L}}{1 \text{ qt}} = 3.752476 \quad \text{round to} \quad 3.75 \text{ L}
\]

Note: Although the calculator gives you a total of 7 places, in CHM 109 you only need to record 2 places beyond the last (farthest to the right) significant place to avoid generating a rounding error. Therefore, instead of writing \(3.752476\) write \(3.75\). In some other classes (physics?) because of the nature of the calculations you might need to record a few more places to avoid making a rounding error, but not in CHM 109.

*From question #2:*

\[
\frac{X}{238 \text{ lb}} = \frac{5.3 \text{ L}}{189.8 \text{ lb}} \quad \Rightarrow \quad X = \frac{5.3 \text{ L} \times 238 \text{ lb}}{189.8 \text{ lb}}
\]

round to

\[
X = 6.645943098 \text{ L} \quad \Rightarrow \quad 6.6 \text{ L}
\]

Although the calculator gave \(6.645943098\) you could have “gotten away with” \(6.646\) (see below).

*From question #3:*

*Done correctly using pre-rounded values:*

\[
\frac{\%}{100} = \frac{\text{part}}{\text{whole}} \quad \Rightarrow \quad \% = \frac{\text{part} \times 100}{\text{whole}} \quad \Rightarrow \quad \% = \frac{3.752476 \text{ L} \times 100}{6.645943098 \text{ L}}
\]

round to

\[
\% = \frac{56.46265616}{100} \quad \Rightarrow \quad 56\%
\]

(continued below)
Done with rounded values:

\[ \% = \frac{3.75 \text{ L} \times 100}{6.6 \text{ L}} = \approx 56.81818182 \text{ rounds to } 57\% \]

Note that the answer is different in a significant place (the ones place) than that obtained above using the pre-rounded values. That is, a rounding error has occurred. Also, the pre-rounded answer differs from the correct pre-rounded answer (see above) in the place right next to the last significant place, the tenths place.

How about trying the calculation using just 2 places past the last significant place?

\[ \% = \frac{3.7525 \text{ L} \times 100}{6.646 \text{ L}} = \approx 56.46253386 \text{ rounds to } 56\% \]

Note that this is the same, correct answer obtained using all places shown by the calculator. The answer obtained with these numbers does not differ from the value obtained using all places given by the calculator until you reach the ten-thousandths place, which is 4 places past the last significant place. Because of the savings in time, wear and tear on your hand/wrist, etc., I recommend writing down just two places past the last significant place when you do these types of problems. Once you become familiar with the rounding rules, it becomes much easier to know which place this is.