The Logic of Quantification

This is a mini-lesson on the use of quantifiers, and some standard symbols for quantifiers, in mathematics.

Basic quantifiers and their symbols
Universal quantifier - "For every" or "For all" $\forall$
Existential quantifier - "There exists" $\exists$

Other common symbols
"Such that" $\ni$
"Implies" $\Rightarrow$
"If… then" $\rightarrow$
"Not" $\sim$ or $\neg$
"If and only if" $\leftrightarrow$

Examples:
Let C be the set of all cats, Y the set of all yellow things, B the set of all blue things, D the set of all dogs.
To write the statement "All cats are yellow", one could write:
$\forall x \ni x \in C, x \in Y.$
To write "Some cats are not yellow", one could write:
$\exists x \ni x \in C$ and x is $\sim Y.$

Practice: Write in symbols
1. "All blue things are dogs."

_____

2. "Some dogs are either blue or yellow."

_____

To negate a universally quantified statement, one uses an existentially quantified statement. To negate an existentially quantified statement, one uses a universally quantified statement.
Example: The negation of "All blue things are dogs" is "Some blue things are not dogs."

Practice: Write, in symbols, the negation of each following statement:
3. "All dogs are yellow."

_____

4. "Some cats are blue."

_____