Fractured Spaghetti and Other Probability Topics

Objectives: To explore the relationship between probability and area,
To approximate probabilities by simulation,
To connect probability and algebra,
To connect probability and geometry.

Materials needed: Uncooked spaghetti,
Phone book or table of random numbers,
Graph paper,
Centimeter Ruler, and at least one of the following:
- Calculator with random number generator (eg TI-82 or -83),
- Computer with X(PLORE),
- Computer with BASIC.

Question: If a piece of spaghetti is broken at two randomly chosen points, what is the probability that the three pieces, placed end-to-end, can form a triangle?

1. We will first approach the problem directly. Each person should do the following: Take a piece of spaghetti 100mm in length. Then, in the phone book or random number table, choose a starting point by closing your eyes and placing your finger on the page. Look at the numbers formed by the right two columns of digits. For example, if your finger hit on the numbers

555-12**12**
567-12**34**

then pick 12 and 34 as your numbers. Pick the first two numbers whose *sum is less than 100*, the length of your spaghetti. [Our purpose here is to choose a *random sample* of two numbers, *i.e.* a sample chosen so that each sample of size two is equally likely to be picked.] Do this <u>ten times</u>, *i.e.* for ten acceptable pairs of numbers. Count the number of times you or your classmates were able to form a triangle with the pieces, and divide by the total number of attempts. [This is called the *relative frequency* of successful attempts.] Record that fraction here: _______________

2. Part 1 was actually an attempt to approximate the **probability** of forming a triangle by randomly breaking spaghetti. We got only ten measurements per person. If we use computer or calculator ***simulation***, we can easily increase the number of trials of this experiment without wasting spaghetti. Each group should select *at least one* of the following simulation procedures. For best opportunity to compare, each procedure should be used at least once by some group.

a. Using X(PLORE), type the following commands on the input area:

```
X =  100 * RANDOM
Y =  100 * RANDOM
Z =  100 - (X+Y)
//  If Z is negative, discard the trial.
// This program assumes a 100 mm stick of spaghetti.
```

Successively invoke each of the top three commands in order by pressing ENTER when the cursor is on the command. If Z is not negative, determine whether a triangle can be formed. (What should you look for to determine if a triangle can be formed?

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____)

If a triangle can be formed with the three lengths, write Y in the following chart. If not write N. Repeat the experiment for at least 40 trials.

Count the number of Y's and divide by the number of trials. This computes the relative frequency approximation to the probability of forming a triangle. Record your fraction here: ___________________ Compare your results with those of other groups.

b. Using the TI-82 or -83, type the following command on the home screen:

```
100 * rand
```

The rand command can be accessed by pressing [MATH], selecting PRB and choosing 1: rand . Once the command is on the screen, press [ENTER] twice. Two numbers between 0 and 100 appear. They represent the lengths of the first two pieces of a broken 100 mm length of spaghetti. If they sum to more than 100, clear the screen and press [ENTER] twice more. If their sum is no more than 100, write them in the following table in the spaces provided and complete the rest of the row of the table, writing a Y in the last column if a triangle can be formed with the resulting number triple. Clear the screen and press [ENTER] twice more to repeat the experiment. Continue until at least 40 trials result in a pair of numbers whose sum is less than 100.

First Number (X)	Second Number (Y)	Third Number 100- (X+Y)	Yes (Y) or No (N)	First Number (X)	Second Number (Y)	Third Number 100- (X+Y)	Yes (Y) or No (N)

After completing the table, calculate the ratio of Yes answers to the total number of trials. This computes the relative frequency approximation to the probability of forming a triangle. Record your fraction here: ___________________ Compare your results with those of other groups.

c. The third simulation method allows us to repeat the experiment in parts a and b thousands of times in a few seconds. It requires a computer with the BASIC programming language. The following program is written for Microsoft BASIC, but can be modified easily for other interpreters (such as Apple BASIC). Simply type the program into the editor and run it. Record your result here: ____________ How does it compare to your previous estimate?

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10  '  This program breaks spaghetti of  length 100mm at two different random points.
20  '  It repeats the experiment for a total of 1000 pieces of spaghetti.
30  '  Then it computes the fraction of times the pieces will form a triangle.
40  '
50  RANDOMIZE
60  C=0
70  YES = 0
80  X = 100 * RND
90  Y = 100 * RND
100  Z = 100 - (X + Y)
110  IF Z  <  0   THEN GOTO  80
120  IF  (X+Y<Z)  OR  (X+Z<Y)  OR  (Y+Z<X)   THEN GOTO 140
130  YES = YES + 1
140  C = C + 1
150  IF  C  <  1000  THEN GOTO  80
160  PRINT  " The ratio of Successes to Total Trials is  "; YES/C
```

3. **The graphical approach**: Suppose, as before, we break a 100mm piece of spaghetti at randomly chosen points so as to form three pieces of lengths X, Y and Z, where $Z = 100 - (X + Y)$. Then the following inequalities hold. Explain why in each case:

a. $X>0$ and $Y>0$: ____

b. $X + Y < 100$: ____

The enclosed region, when these inequalities are graphed, forms the set of all points (X,Y) that are possible lengths of broken pieces of spaghetti. In other words, this region is our domain or sample space.

If, in addition, the three pieces form a triangle, the following are true. Explain why and then simplify each expression:

c. X + Y > 100 - (X + Y). Why?____

Simple form: ______________________________

d. X + [100 - (X + Y)] > Y. Why? ____

Simple form: ______________________________

e. Y + [100 - (X + Y)] > X . Why? ____

Simple form: ______________________________

On your graph paper, graph **all six** of the simplified inequalities on the **same set of axes** and shade the portion corresponding to the formation of a triangle. What fraction of total area (sample space) is covered by the shaded portion? ______ How does it compare to the results you got by simulation?

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Recall our definition of the probability of an event as the expected long term relative frequency of occurrence of that event. Which of the three above approaches to finding the probability of triangle construction do you think is most accurate and why? (The three approaches are the phone book, computer simulation and graphical analysis.)

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