

# Puff the Cubic Function

(How many puffs are needed to expand a balloon to a given diameter?)

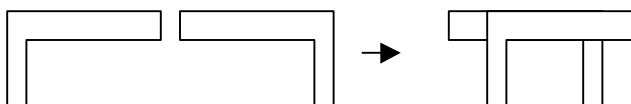
- Materials:
1. A large spherical party balloon
  2. A means of measuring balloon diameter (e.g. 2 carpenter squares. See below.)
  3. A calculator or computer with linear regression capability

Introduction: You have learned that your calculator or computer spreadsheet can easily produce a function or model of the form  $y = ax + b$  if you supply it with a sufficient number of pairs (x,y) of data. In the Data Transformation section of your booklet, you find that you can use the linear regression capability of your technology to produce nonlinear functions. For example, if you wish to produce a quadratic function of the form  $y = ax^2 + b$ , you need simply to supply your calculator or spreadsheet with data in the form (x<sup>2</sup>,y). It will dutifully produce a “slope” value for  $a$  and an intercept value for  $b$  to complete the model. Just remember that the model is  $y = ax^2 + b$ , not  $y = ax + b$ . Likewise, if you wish to produce a model of the form  $y = ax^3 + b$ , supply your linear regression tool with data pairs of the form (x<sup>3</sup>,y).

Procedure for data collection: The instructor (if this is an individual activity) or the group coordinator (if this is a group activity) will do the following: First practice blowing up your balloon with large puffs of uniform size. This relaxes the balloon and helps ensure that equal amounts of air are contained in each puff. Then, with the help of a colleague, measure the diameter  $x$  of the balloon after  $y = 1$  puff. Record these values in the chart below. Without allowing the balloon to deflate, add one or two more puffs of uniform size. Measure the diameter and record the data pair below. Continue this procedure to obtain six or seven data pairs.

Diameter (x)	No. of Puffs (y)	Transformed data (x <sup>3</sup> )
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

Carpenter Square Calipers:



The problem: Our objective is to produce a function that will predict the number of puffs (**y**) that will produce a balloon of diameter **x**. Since the diameter (**x**) has a linear measure (e.g. inches) and the volume of air produced by **y** puffs has cubic measure (e.g. cubic inches), we will choose a function of the form  $y = ax^3 + b$ .

Question 1: Why do we choose this cubic and not the more general  $y = ax^3 + bx^2 + cx + d$ ? Considering the formula for the volume of a sphere may help you with the answer.

Now pick one data pair from the middle of your data table and eliminate it from your list. Write it here: (\_\_\_\_, \_\_\_\_). Supply the remaining data to your linear regression tool and produce the number of puffs **y** needed for a balloon of diameter **x**:

$$y = \text{_____} x^3 + \text{_____} .$$

Use your model to answer the following questions:

2. What does your model predict the number of puffs should be for the diameter in your omitted point above? \_\_\_\_\_. Show or describe your calculations:
  
3. What does your model predict the diameter should be for the number of puffs in your omitted point above? \_\_\_\_\_. Show or describe your calculations:
  
4. The y-intercept of your model is very likely a positive number. Why would one expect this to be true?
  
5. From the list below, choose the model you think will best describe:
  - a. the circumference of a tree (y) as a function of its diameter (x): \_\_\_\_\_
  - b. The number of drops of water (y) needed to produce a circular puddle of diameter (x): \_\_\_\_\_

Be prepared to justify your choices in class discussion.

- (i)  $y = ax + b$    (ii)  $y = ax$    (iii)  $y = ax^2$    (iv)  $y = ax^2 + b$    (v)  $y = a(1/x) + b$   
(vi)  $y = ax^2 + bx + c$    (vii)  $y = ax^3$    (viii) Other, specify:  $y = \text{_____}$