

# Age of Menarche

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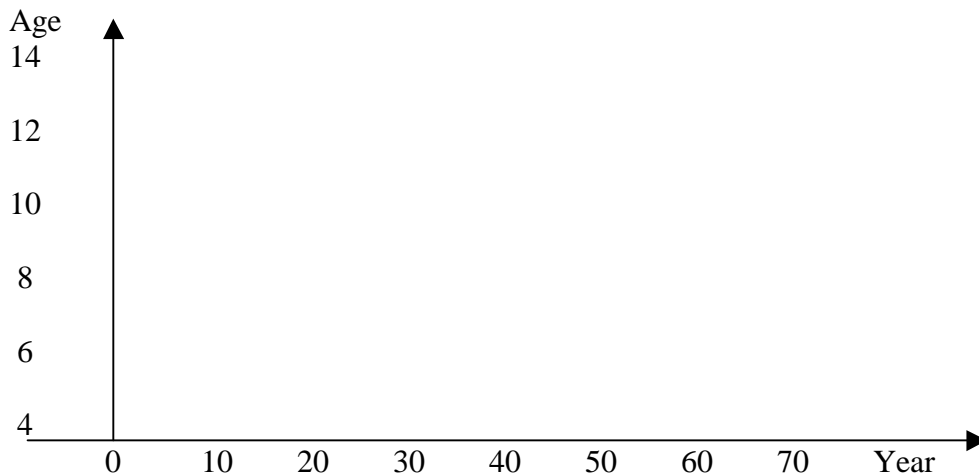
A psychology class at the university had been studying the effects of nutrition on human growth and development. Members of the class were especially interested in the progression over time of the average age of menarche - the age at which young females have their first menstruation.

James, a psychology major in the class, was also enrolled in College Mathematics and had just studied modeling with the exponential function. He saw a connection between the classes. James wrote to his math professor, "I learned that as time increased in years, that the age at which females got their first period (or reach puberty) decreased. I also knew that this age had to level off at some point because the amount of fat necessary for menarche had to be accumulated over time. I don't think a baby could be born with enough body fat to sustain the effects of puberty. I know there are other factors to consider, but there must be a minimum age at which the body can have enough fat to sustain menarche. I was presented with data like this in class:

<u>AverageAge of Menarche</u>	<u>Year</u>
14	1900
13	1907
12	1915
11	1918
10	1925
9	1950

I see from a graph of the data that it is shaped like an exponential function. The only problem is that all exponentials have the x-axis as their horizontal asymptote. If I can find a function type that has the same shape but with a non-zero horizontal asymptote, I can fit it to the data and find the lowest value (age) at which the body can sustain menarche."

Part I: With "year" as your independent variable and "age" as your dependent variable, graph the data and draw a horizontal line that you think represents the asymptote James wanted to find. Code year 1900 as your zero year so that the year 1950 is represented simply as 50.



James went on to say, "Another problem is that I don't know how to fit the data to a function type that is not on my calculator's list of regression functions - unless I transform the data somehow. Pretty obviously, I've got to start with the exponential function."

Part II:

1. Which of the following transformations of the exponential function  $Y = a \cdot b^x$  will produce a function shaped like the data graphed in Part I? (Hint: Which transformation will "lift the exponential off the x-axis"?) Circle one.

(a)  $Y = a \cdot b^{x-c}$       (c)  $Y = a \cdot b^{x+c}$       (c)  $Y = a \cdot b^x + c$       (d)  $Y = a \cdot b^{cx}$

2. Judging from your graph in Part I, what do you expect the value of the parameter "c" to be in your model? \_\_\_\_\_. Explain why you think "c" is that value:

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Now back to our story. James came to talk with the prof. "O.K., so I choose a value of "c" and I subtract it from all the values of Y in my data. The new data set will be a set of pairs (x, y-c) whose graph will be the same shape as the original graph, but the new graph will skirt along the x-axis - I hope. I can use exponential modeling to get a function that fits the new data."

Part III: Using the value of "c" you chose in Part II, no.2, carry out James' plan. Subtract your value of "c" from each y-value and fit an exponential function to the resulting data pairs. Write the answer from your calculator here:

$Y =$  \_\_\_\_\_

and write the corresponding function that fits the original data here:

$Y =$  \_\_\_\_\_.

James continued, "But how do I tell a good guess for 'c' from a bad one? I need some way to tell if my guess is good."

"There is no single best way to make that judgement," replied the prof. "But you have the correlation coefficient 'r', or its associated coefficient of determination 'r-square' that will help you judge how well the model fits the data set. And your TI-83 gives you both of them when you choose the exponential regression model from the Stat|Calc menu\*. Why don't you refine your guess for 'c' until r-square gets very close to one (1). You may have to make several educated guesses at 'c'."

\* If r-square does not display with your regression function, go to Catalog and execute DiagnosticOn.

Part IV: Take the profs advice. Plan a good strategy for improving your guess at the parameter "c" and carry out your strategy until you are satisfied that the value of "c" is correct to one decimal place. Explain your plan here:

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Now record the results of your regression.

Last improvement of your function as displayed on the calculator:

$$Y = \underline{\hspace{2cm}}$$

Last improvement of your function that fits the original data:

$$Y = \underline{\hspace{2cm}}$$

Write a short report that explains your final results. Be sure to explain what your findings say about human growth and development. Attach your word-processed report after this page.