

1. Integration by Substitution

We know these integral rules:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \text{ where } C \text{ is any number.}$$

$$\int e^x dx = e^x + C \text{ and}$$

$$\int \frac{1}{x} dx = \ln(x) + C.$$

So we also know these rules:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \text{ where } C \text{ is any number.}$$

$$\int e^u du = e^u + C \text{ and}$$

$$\int \frac{1}{u} du = \ln(u) + C.$$

Let's try an integral like

$$\int (x^2 + 1)^3 2x dx.$$

First let $u = x^2 + 1$ so that $du/dx = 2x$. That makes $du = 2x dx$.

Making these substitutions, we get

$$\int (x^2 + 1)^3 2x dx = \int u^3 du$$

which gives us

$$\frac{u^4}{4} + C.$$

Replacing u with $x^2 + 1$ then gives

$$\frac{(x^2+1)^4}{4} + C.$$

Now try these:

1. $\int (x^3 + 1)^4 3x^2 dx =$

2. $\int (x^3 + 1)^4 x^2 dx =$

3. $\int (x^3 + 1)^4 5x^2 dx =$

4. $\int e^{x^2+1} x dx =$

5. $\int \frac{2x}{x^2+1} dx =$

6. $\int (x^2 + x)^3 (2x + 1) dx =$