

## Hypothesis Testing - verP

[The following summary details how to conduct a test to reject a null hypothesis based on information in a simple random sample. As an option to rejecting the null hypothesis, one can simply report the "p-value". **The "p-value" is the probability that, if the null hypothesis is true, we still get results as extreme as those we obtained from the sample.**]

### Testing Claims about a Population Mean $\mu$ with $\sigma$ Known

Assumptions:

1. Observations are from a simple random sample.
2. The value of the population standard deviation  $\sigma$  is known.
3. The population is Normally distributed OR  $n > 30$  OR both.

Format of an hypothesis test on  $\mu$ :

$H_0: \mu = \mu_0$  ( $\mu$  is some specified value.)

- Ha:
- (1)  $\mu > \mu_0$  (upper tailed test)
  - (2)  $\mu < \mu_0$  (lower tailed test)
  - (3)  $\mu \neq \mu_0$  (two tailed test)

Test Statistic:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Rejection Criteria: Reject  $H_0$  with probability of false rejection  $\alpha$  under the following conditions ( $z_\alpha$  is the value of the standard normal with table value  $1-\alpha$ ):

1. For an upper tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z > z_\alpha$ ).
2. For a lower tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z < -z_\alpha$ ).
3. For a two tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z > z_{\frac{\alpha}{2}}$  or  $z < -z_{\frac{\alpha}{2}}$ ).

## Testing Claims about a Population Mean $\mu$ with $\sigma$ NOT Known

Assumptions:

1. Observations are from a simple random sample.
2. The value of the population standard deviation  $\sigma$  is not known.
3. The population is Normally distributed OR  $n > 30$  OR both.

Format of an hypothesis test on  $\mu$ :

$H_0: \mu = \mu_0$  ( $\mu$  is some specified value.)

- $H_a:$
- (1)  $\mu > \mu_0$  (upper tailed test)
  - (2)  $\mu < \mu_0$  (lower tailed test)
  - (3)  $\mu \neq \mu_0$  (two tailed test)

Test Statistic:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Rejection Criteria: Reject  $H_0$  with probability of false rejection  $\alpha$  under the following conditions ( $t_{\alpha, n-1}$  is the value of the t distribution with  $n-1$  degrees of freedom and with area in one tail =  $\alpha$ ):

1. For an upper tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $t > t_{\alpha, n-1}$ ).
2. For a lower tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $t < -t_{\alpha, n-1}$ ).
3. For a two tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $t > t_{\frac{\alpha}{2}, n-1}$  or  $t < -t_{\frac{\alpha}{2}, n-1}$ ).

## Testing Claims about a Population Proportion $p$

Assumptions:

1. Observations are from a simple random sample of  $n$  independent trials, each with two outcomes called Success and Failure.
2. The Probability of Success  $p$  is constant from trial to trial.
3. The quantities  $np$  and  $nq$  are each at least 5, where  $q=1-p$ .
4. The value of  $p$  is estimated from the sample by the quantity

$$\hat{p} = \frac{\text{number of successes}}{\text{number of trials}}.$$

Format of an hypothesis test on  $p$ :

$H_0: p = p_0$  ( $p$  is some specified value.)

$H_a:$  (1)  $p > p_0$  (upper tailed test)  
(2)  $p < p_0$  (lower tailed test)  
(3)  $p \neq p_0$  (two tailed test)

Test Statistic:

$$z = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0 q_0}{n}}}$$

Rejection Criteria: Reject  $H_0$  with probability of false rejection  $\alpha$  under the following conditions ( $z_\alpha$  is the value of the standard normal with table value  $1-\alpha$ ):

- (1) For an upper tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z > z_\alpha$ ).
- (2) For a lower tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z < -z_\alpha$ ).
- (3) For a two tailed test, reject  $H_0$  if the p-value is less than  $\alpha$  (or, equivalently, if  $z < -z_{\alpha/2}$  or if  $z > z_{\alpha/2}$ ).