

Confidence Interval Estimation - Ver B

Estimating a Population Mean μ when σ is Known

Assumptions:

1. Observations are from a simple random sample.
2. The value of the population standard deviation σ is known.
3. The population is Normally distributed OR $n > 30$ OR both.

Then the $(1-\alpha)$ confidence interval for μ is:

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

As for any of our confidence intervals based on the Normal,

For 90% confidence, use $z_{\alpha/2} = 1.645$;

For 95% confidence, use $z_{\alpha/2} = 1.96$; and

For 99% confidence, use $z_{\alpha/2} = 2.576$.

Estimating a Population Mean μ with σ NOT Known

Assumptions:

1. Observations are from a simple random sample.
2. The value of the population standard deviation σ is not known.
3. The population is Normally distributed OR $n > 30$ OR both.

Then the $(1-\alpha)$ confidence interval for μ is:

$$\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

In this case, the value of $t_{\alpha/2}$ must be determined from the Student t Table using the "two-tailed" value for the confidence level wanted and using $n-1$ degrees of freedom.

Estimating a Population Proportion p

Assumptions:

1. Observations are from a simple random sample of n independent trials, each with two outcomes called Success and Failure.
2. The Probability of Success p is constant from trial to trial.
3. The quantities np and nq are each at least 5, where $q=1-p$.
4. The value of p is estimated from the sample by the quantity

$$\hat{p} = \frac{\text{number of successes}}{\text{number of trials}}.$$

Then the $(1-\alpha)$ confidence interval for p is:

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p} \hat{q}}{n}} < p < \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p} \hat{q}}{n}}$$

As for any of our confidence intervals based on the Normal,

For 90% confidence, use $z_{\alpha/2} = 1.645$;

For 95% confidence, use $z_{\alpha/2} = 1.96$; and

For 99% confidence, use $z_{\alpha/2} = 2.576$.